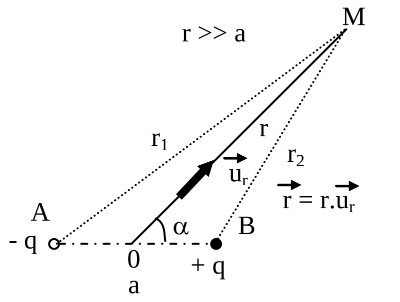
POTENTIEL CREE PAR UN DIPOLE

- Potentiel créé en M par la charge –q placée en A : $V_1^{(A)} = \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_1}$
- Potentiel créé en M par la charge +q placée en B : $V_2^{(B)} = \frac{1}{4\pi\varepsilon_0}\frac{+q}{r_2}$
- Potentiel créé en M par le dipôle : $V_M = V_1^{(A)} + V_2^{(B)} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$



- Relation de Pythagore pour un triangle quelconque
- Triangle AOM $\quad r_1^2 = \left(\frac{a}{2}\right)^2 + r^2 - 2.\frac{a}{2}.r.\cos(\pi-\alpha) = r^2\left(1+\frac{a^2}{4r^2}+\frac{a.\cos\alpha}{r}\right) \Rightarrow r_1 = r\left(1+\frac{a^2}{4r^2}+\frac{a.\cos\alpha}{r}\right)^{\frac{1}{2}}$
- Triangle BOM $\quad r_2^2 = \left(\frac{a}{2}\right)^2 + r^2 - 2.\frac{a}{2}.r.\cos\alpha = r^2\left(1+\frac{a^2}{4r^2}-\frac{a.\cos\alpha}{r}\right) \Rightarrow r_2 = r\left(1+\frac{a^2}{4r^2}-\frac{a.\cos\alpha}{r}\right)^{\frac{1}{2}}$

On suppose r >> a donc en effectuant un développement limité au 1e ordre sachant que $(1+\varepsilon)^n = 1+n.\varepsilon$, on obtient :

$$\frac{1}{r_1} = r_1^{-1} = r^{-1}\left(1+\frac{a^2}{4r^2}+\frac{a.\cos\alpha}{r}\right)^{-\frac{1}{2}} = r^{-1}\left(1-\frac{a^2}{8r^2}-\frac{a.\cos\alpha}{2r}\right)$$

$$\frac{1}{r_2} = r_2^{-1} = r^{-1}\left(1+\frac{a^2}{4r^2}-\frac{a.\cos\alpha}{r}\right)^{-\frac{1}{2}} = r^{-1}\left(1-\frac{a^2}{8r^2}+\frac{a.\cos\alpha}{2r}\right)$$

$$V_M = V_1^{(A)} + V_2^{(B)} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = \frac{q}{4\pi\varepsilon_0}r^{-1}\left(2\frac{a.\cos\alpha}{2r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{p.\cos\alpha}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\vec{u}_r}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\vec{r}}{r^3}$$

$$\boxed{V_M = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\vec{r}}{r^3}}$$

EXPRESSION DU CHAMP ELECTRIQUE CREE PAR UN DIPOLE

$\vec{E} = -\overrightarrow{grad}\,V$ les coordonnées polaires sont les plus appropriées $\overrightarrow{grad}\left(\frac{\partial}{\partial r}, \frac{\partial}{r.\partial\alpha}\right)$

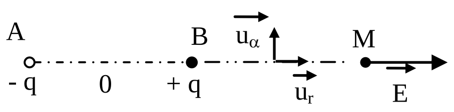
$$E_r = -\frac{\partial}{\partial r}V = -\frac{\partial}{\partial r}\left(\frac{p.\cos\alpha}{4\pi\varepsilon_0 r^2}\right) = \frac{2p.\cos\alpha}{4\pi\varepsilon_0 r^3} = \frac{2\vec{p}.\vec{r}}{4\pi\varepsilon_0 r^4}$$

$$E_\alpha = -\frac{\partial}{r.\partial\alpha}V = -\frac{\partial}{r.\partial\alpha}\left(\frac{p.\cos\alpha}{4\pi\varepsilon_0 r^2}\right) = \frac{p.\sin\alpha}{4\pi\varepsilon_0 r^3} = \frac{\vec{p}\wedge\vec{r}}{4\pi\varepsilon_0 r^4}$$

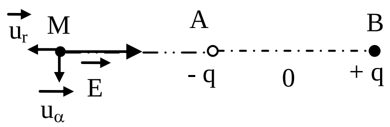
$$E = \sqrt{E_r^2 + E_\alpha^2} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4.\cos^2\alpha + \sin^2\alpha} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3.\cos^2\alpha + 1}$$

- **<u>E maximum</u>** quand α = 0 ou π.

$$\alpha = 0 \quad \Rightarrow \quad E = E_r = \frac{2p}{4\pi\varepsilon_0 r^3}$$



$$\alpha = \pi \quad \Rightarrow \quad E = E_r = \frac{-2p}{4\pi\varepsilon_0 r^3}$$



- **<u>E minimum</u>** quand **α = π/2 ou -π/2.**

$$\alpha = \frac{\pi}{2} \quad \Rightarrow \quad E = E_\alpha = \frac{p}{4\pi\varepsilon_0 r^3}$$

$$\alpha = -\frac{\pi}{2} \quad \Rightarrow \quad E = E_\alpha = \frac{-p}{4\pi\varepsilon_0 r^3}$$